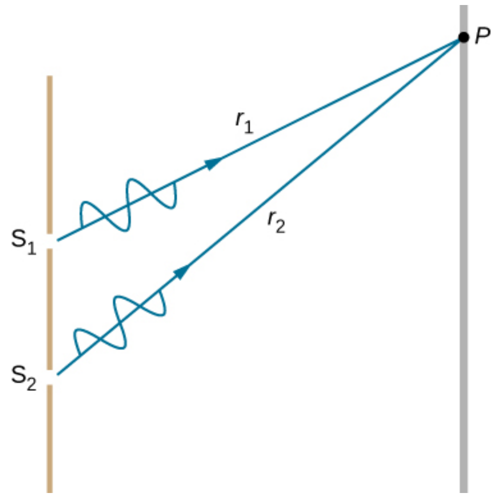

Figure 3.6 Waves follow different paths from the slits to a common point *P* on a screen. Destructive interference occurs where one path is a half wavelength longer than the other—the waves start in phase but arrive out of phase. Constructive interference occurs where one path is a whole wavelength longer than the other—the waves start out and arrive in phase.

3.2 | Mathematics of Interference

Learning Objectives

By the end of this section, you will be able to:

- Determine the angles for bright and dark fringes for double slit interference
- Calculate the positions of bright fringes on a screen

Figure 3.7(a) shows how to determine the path length difference Δl for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen [part (b)] is nearly the same for each path. In other words, r_1 and r_2 are essentially parallel. The lengths of r_1 and r_2 differ by Δl, as indicated by the two dashed lines in the figure. Simple trigonometry shows

$$\Delta l = d \sin \theta \tag{3.3}$$

where *d* is the distance between the slits. Combining this result with **Equation 3.1**, we obtain constructive interference for a double slit when the path length difference is an integral multiple of the wavelength, or

$$d \sin \theta = m\lambda, \text{ for } m = 0, \pm1, \pm2, \pm3,\ldots \text{ (constructive interference)}. \tag{3.4}$$

Similarly, to obtain destructive interference for a double slit, the path length difference must be a half-integral multiple of the wavelength, or

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \text{ for } m = 0, \pm1, \pm2, \pm3,\ldots \text{ (destructive interference)} \tag{3.5}$$

where λ is the wavelength of the light, *d* is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call *m* the **order** of the interference. For example, $m = 4$ is fourth-order interference.

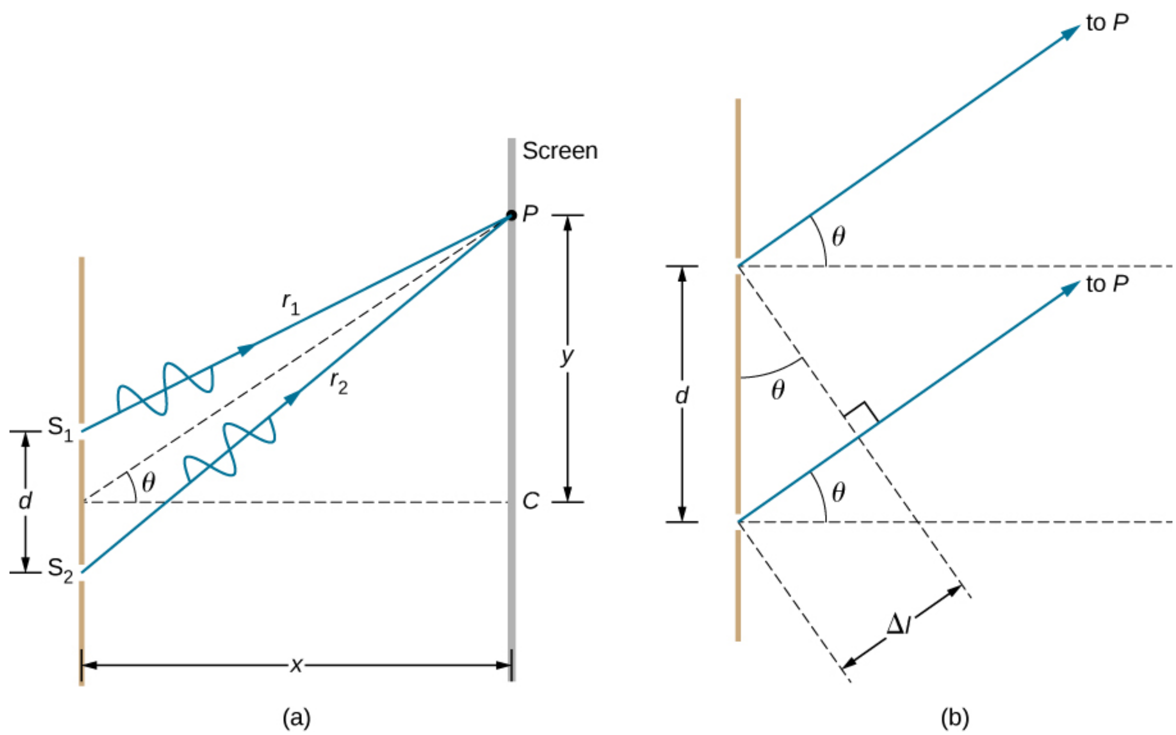

Figure 3.7 (a) To reach P, the light waves from S_1 and S_2 must travel different distances. (b) The path difference between the two rays is Δl.

The equations for double-slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on either side of the incident beam into a pattern called interference **fringes** (**Figure 3.8**). The closer the slits are, the more the bright fringes spread apart. We can see this by examining the equation $d \sin\theta = m\lambda$, for $m = 0, \pm1, \pm2, \pm3\ldots$. For fixed λ and m, the smaller d is, the larger θ must be, since $\sin\theta = m\lambda/d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large θ, hence, a large effect.

Referring back to part (a) of the figure, θ is typically small enough that $\sin\theta \approx \tan\theta \approx y_m/D$, where y_m is the distance from the central maximum to the mth bright fringe and D is the distance between the slit and the screen. **Equation 3.4** may then be written as

$$d\frac{y_m}{D} = m\lambda$$

or

$$y_m = \frac{m\lambda D}{d}. \tag{3.6}$$

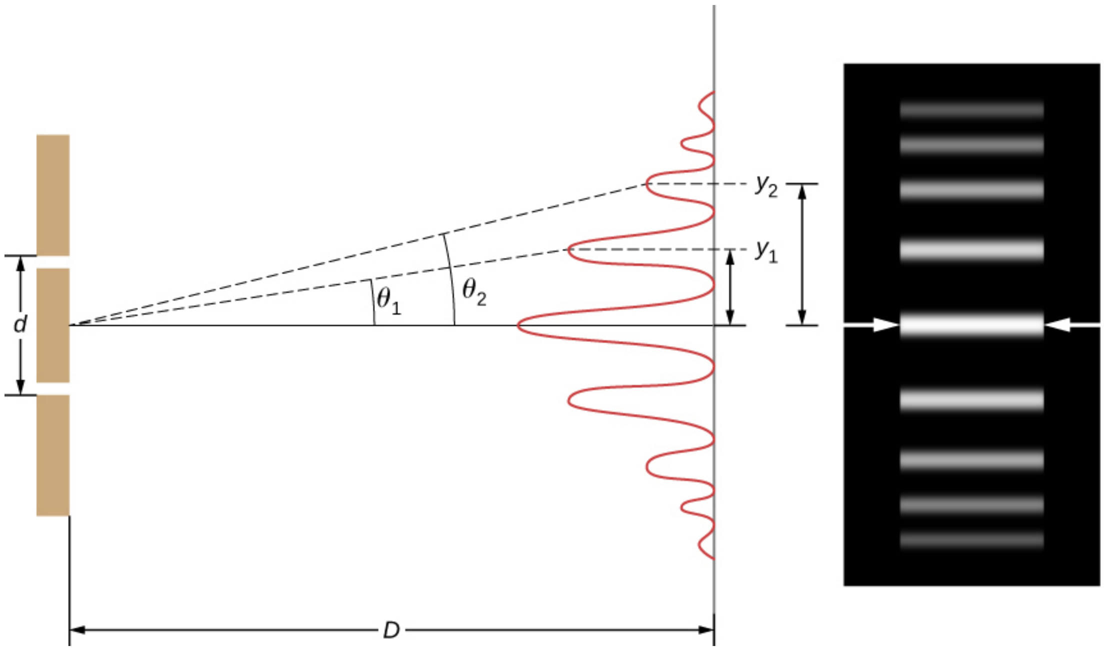

Figure 3.8 The interference pattern for a double slit has an intensity that falls off with angle. The image shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Example 3.1

Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of $10.95°$ relative to the incident beam. What is the wavelength of the light?

Strategy

The phenomenon is two-slit interference as illustrated in **Figure 3.8** and the third bright line is due to third-order constructive interference, which means that $m = 3$. We are given $d = 0.0100\ \text{mm}$ and $\theta = 10.95°$. The wavelength can thus be found using the equation $d \sin \theta = m\lambda$ for constructive interference.

Solution

Solving $d \sin \theta = m\lambda$ for the wavelength λ gives

$$\lambda = \frac{d \sin \theta}{m}.$$

Substituting known values yields

$$\lambda = \frac{(0.0100\ \text{mm})(\sin 10.95°)}{3} = 6.33 \times 10^{-4}\ \text{mm} = 633\ \text{nm}.$$

Significance

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical techinque is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ , so that spectra (measurements of intensity versus wavelength) can be obtained.

Example 3.2

Calculating the Highest Order Possible

Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

Strategy

The equation $d \sin \theta = m\lambda$ (for $m = 0, \pm 1, \pm 2, \pm 3\ldots$) describes constructive interference from two slits. For fixed values of d and λ, the larger m is, the larger $\sin \theta$ is. However, the maximum value that $\sin \theta$ can have is 1, for an angle of $90°$. (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find what value of m corresponds to this maximum diffraction angle.

Solution

Solving the equation $d \sin \theta = m\lambda$ for m gives

$$m = \frac{d \sin \theta}{\lambda}.$$

Taking $\sin \theta = 1$ and substituting the values of d and λ from the preceding example gives

$$m = \frac{(0.0100 \text{ mm})(1)}{633 \text{ nm}} \approx 15.8.$$

Therefore, the largest integer m can be is 15, or $m = 15$.

Significance

The number of fringes depends on the wavelength and slit separation. The number of fringes is very large for large slit separations. However, recall (see **The Propagation of Light** and the introduction for this chapter) that wave interference is only prominent when the wave interacts with objects that are not large compared to the wavelength. Therefore, if the slit separation and the sizes of the slits become much greater than the wavelength, the intensity pattern of light on the screen changes, so there are simply two bright lines cast by the slits, as expected, when light behaves like rays. We also note that the fringes get fainter farther away from the center. Consequently, not all 15 fringes may be observable.

3.1 Check Your Understanding In the system used in the preceding examples, at what angles are the first and the second bright fringes formed?

3.3 | Multiple-Slit Interference

Learning Objectives

By the end of this section, you will be able to:

- Describe the locations and intensities of secondary maxima for multiple-slit interference

Analyzing the interference of light passing through two slits lays out the theoretical framework of interference and gives us a historical insight into Thomas Young's experiments. However, much of the modern-day application of slit interference uses not just two slits but many, approaching infinity for practical purposes. The key optical element is called a diffraction grating, an important tool in optical analysis, which we discuss in detail in **Diffraction**. Here, we start the analysis of multiple-slit interference by taking the results from our analysis of the double slit ($N = 2$) and extending it to configurations with three, four, and much larger numbers of slits.

Figure 3.9 shows the simplest case of multiple-slit interference, with three slits, or $N = 3$. The spacing between slits is d, and the path length difference between adjacent slits is $d \sin \theta$, same as the case for the double slit. What is new is that the path length difference for the first and the third slits is $2d \sin \theta$. The condition for constructive interference is the same as